Current Distribution Analysis for a Multilayer High-Tc Superconducting Cable Considering Magnetic Hysteresis

Weijie Xu[1,2], Nana Duan[1,2], Shuhong Wang[1], Jianguo Zhu[2] and Youguang Guo[2]

[1]Faculty of Electrical Engineering, Xi'an Jiaotong University, 28 West Xianning Rd, Xi'an 710049, China,
shwang@mail.xjtu.edu.cn
[2]School of Electrical, Mechanical and Mechatronic Systems, University of Technology, Sydney, NSW 2007, Australia,
duannana@outlook.com

This paper presents a new Preisach type hysteresis model for the high temperature superconductor. This model requires only the limiting hysteresis loop as the input data, and for this model, the limiting hysteresis loop is firstly separated into two limiting *M−H* loops based on the mechanisms, which can then be modeled by two separate modified Preisach algorithms. The area integrations of the Preisach distribution functions are determined only based on the limiting *M−H* loops. The nonlinear dynamic circuit model of the superconductor is established. In the circuit model, the hysteresis inductance and hysteresis loss described by using the new Preisach type model are deduced. Applying the hysteresis circuit model, the currents flowing in different superconductor layers of high temperature superconducting (HTS) cable are simulated, as well as the hysteresis loss of the superconducting cable. The simulation results are verified by comparison with the data recorded in literatures. Finally, the influences of hysteresis on superconducting cable are analyzed and discussed.

***Index Terms*— Current distribution, equivalent circuit, magnetic hysteresis, superconducting cable.**

I. INTRODUCTION

HIGH TEMPERATURE superconducting (HTS) cables for large current transmission in general have a multilayer structure consisting of parallel connected tapes, twisted in each layer. The control of current distribution among these layers is an important issue for design and optimization of a HTS cable because this would significantly affect the current transmission capacity and power losses. The pinned magnetic flux constitutes a memory that gives rise to a hysteresis loss and produces an additional voltage in the HTS cable [1, 2].

A basic property of the superconducting material is the *B−H* relationship, which can be well described by Bean's critical-state model [3]. However, it has been gradually realized that the critical-state models have some intrinsic limitations owing to the fact that the explicit analytical solutions of the field equations are only possible for samples of very simple geometry. The classical Preisach model is applicable to describe the hysteresis phenomena of the superconducting samples of complex geometry [4]. However, the distribution function is very difficult to determine. The statistic method or a set of first and second order transition curves are usually required according to a number of repeatable experiments.

II. NEW PREISACH TYPE MODEL

Based on the mechanisms of HTS, a new Preisach type HTS hysteresis model is proposed which requires only the limiting hysteresis loop as the input data. In this model, the limiting hysteresis loop is firstly separated into two limiting *M−H* loops as shown in Fig. 1, which can then be modeled by two separate modified Preisach algorithms. The area integrations of the Preisach distribution functions are determined only based on the limiting *M−H* loops [5].

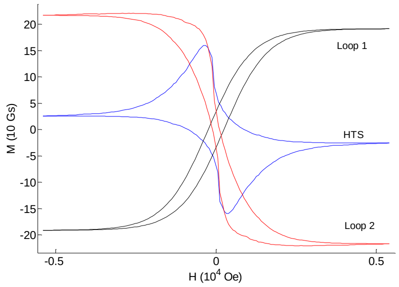


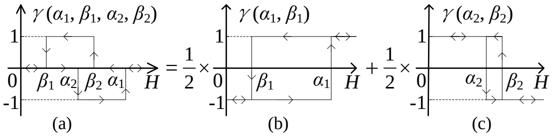
Fig. 1. Separation of limiting *M-H* loop.

The new Preisach type theory describes the HTS hysteresis via an infinite set of magnetic operators, which have elementary hysteresis loops of different switching values of magnetic field strength (α_1, β_1, α_2, β_2), as shown in Fig. 2(a), where α_1 and α_2 are the magnetic field strength in the increasingly positive direction, β_1 and β_2 in the negative direction. This operator can be separated into paramagnetic operator and antimagnetic operator, as shown in Fig. 2(b) and (c), respectively, based on the mechanisms of HTS. The magnetisation *M* induced by an applied magnetic field *H* can be expressed as

$$\begin{aligned} M &= \int_{\mathrm{S}} \mu(\alpha,\beta)\gamma_{\alpha\beta}(H)d\alpha d\beta \\ &= \int_{\mathrm{S_p}} \mu_{\mathrm{p}}(\alpha,\beta)\gamma_{\alpha\beta}(H)d\alpha d\beta + \int_{\mathrm{S_a}} \mu_{\mathrm{a}}(\alpha,\beta)\gamma_{\alpha\beta}(H)d\alpha d\beta \\ &= \int_{\mathrm{S^+}} \mu(\alpha,\beta)\gamma_{\alpha\beta}(H)d\alpha d\beta - \int_{\mathrm{S^-}} \mu(\alpha,\beta)\gamma_{\alpha\beta}(H)d\alpha d\beta \\ &= \int_{\mathrm{S_p^+}} \mu_{\mathrm{p}}(\alpha,\beta)\gamma_{\alpha\beta}(H)d\alpha d\beta - \int_{\mathrm{S_p^-}} \mu_{\mathrm{p}}(\alpha,\beta)\gamma_{\alpha\beta}(H)d\alpha d\beta + \\ &\quad \int_{\mathrm{S_a^+}} \mu_{\mathrm{a}}(\alpha,\beta)\gamma_{\alpha\beta}(H)d\alpha d\beta - \int_{\mathrm{S_a^-}} \mu_{\mathrm{a}}(\alpha,\beta)\gamma_{\alpha\beta}(H)d\alpha d\beta. \end{aligned} \tag{1}$$

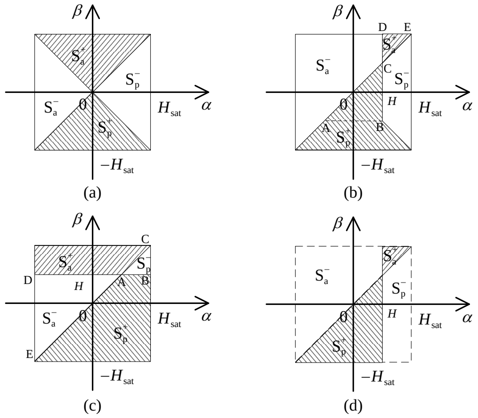
where $\mathrm{S}=\mathrm{S_p}\cup\mathrm{S_a}$, $\mathrm{S}^+=\mathrm{S_p}^+\cup\mathrm{S_a}^+$, $\mathrm{S}^-=\mathrm{S_p}^-\cup\mathrm{S_a}^-$, $\mu(\alpha, \beta)=\mu_{\mathrm{p}}(\alpha, \beta)\cup\mu_{\mathrm{a}}(\alpha, \beta)$. S is the rectangular region $H_{\mathrm{sat}}>\alpha>-H_{\mathrm{sat}}$, $H_{\mathrm{sat}}>\beta>-H_{\mathrm{sat}}$ on the (α, β) plane, as shown in Fig. 3, known as the modified Preisach diagram, where $\mathrm{S_p}$ and $\mathrm{S_a}$ are the lower and upper triangular regions $H_{\mathrm{sat}}>\alpha>\beta>-H_{\mathrm{sat}}$, $H_{\mathrm{sat}}>\beta>\alpha>-H_{\mathrm{sat}}$, $\mu_{\mathrm{p}}(\alpha, \beta)$ and

$\mu_a(\alpha, \beta)$ the distribution functions of the paramagnetic and antimagnetic operators, $\mu(\alpha, \beta)=0$ if $(\alpha, \beta)\notin S$, $\gamma_{\alpha\beta}(H)=1$ on S^+, $\gamma_{\alpha\beta}(H)=-1$ on S^-. H_{sat} is the saturation magnetic field strength. Subscript p denotes the paramagnetic part, and subscript a the antimagnetic part.



(a) (b) (c)

Fig. 2 Elementary hysteresis loop of operators.



(a) (b) (c) (d)

Fig. 3 New Preisach type diagrams (a) unmagnetised state, (b) when magnetised along the initial magnetisation curve, (c) on the downward trajectory of the limiting loop, and (d) on the upward trajectory of the limiting loop.

III. NONLINEAR DYNAMIC CIRCUIT MODEL CONSIDERING MAGNETIC HYSTERESIS

The nonlinear dynamic circuit model considering hysteresis inductance deduced according to this new model is shown in Fig. 4. Applying this equivalent circuit, the currents flowing in different superconductor layers of HTS cable are simulated, as well as the hysteresis loss of the superconducting cable. The simulation results verified by comparison with the data recorded in literatures [6].

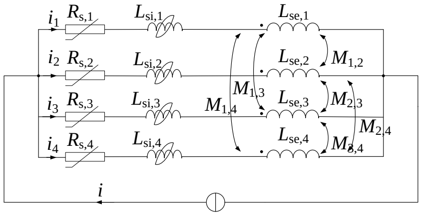


Fig. 4. Equivalent circuit of HTS cable considering hysteresis.

The determination of self, mutual inductances and nonlinear resistances superconducting layers may refer to [6].

The circuit equations can be deduced as

$$\begin{cases} R_{s,1}i_1(t)+L_{1,1}\frac{di_1(t)}{dt}+L_{h1,1}\frac{di_1(t)}{dt}+L_{1,2}\frac{di_2(t)}{dt}+L_{1,3}\frac{di_3(t)}{dt}+L_{1,4}\frac{di_4(t)}{dt}=u(t) \\ L_{2,1}\frac{di_1(t)}{dt}+R_{s,2}i_2(t)+L_2\frac{di_2(t)}{dt}+L_{h2,2}\frac{di_2(t)}{dt}+M_{2,3}\frac{di_3(t)}{dt}+M_{2,4}\frac{di_4(t)}{dt}=u(t) \\ M_{3,1}\frac{di_1(t)}{dt}+M_{3,2}\frac{di_2(t)}{dt}+R_{s,3}i_3(t)+L_3\frac{di_3(t)}{dt}+L_{h3,3}\frac{di_3(t)}{dt}+M_{3,4}\frac{di_4(t)}{dt}=u(t) \\ M_{4,1}\frac{di_1(t)}{dt}+M_{4,2}\frac{di_2(t)}{dt}+M_{4,3}\frac{di_3(t)}{dt}+R_{s,4}i_4(t)+L_4\frac{di_4(t)}{dt}+L_{h4,4}\frac{di_4(t)}{dt}=u(t) \\ i_1(t)+i_2(t)+i_3(t)+i_4(t)=i(t) \end{cases} \quad (2)$$

where L_h is the hysteresis inductance calculated by using the new Preisach type model. These nonlinear differential equations may be solved by using Newton method.

IV. NUMERICAL EXAMPLE

In order to verify the correction of the new Preisach type model, the superconducting cable prototype, which is simulated in [6], is employed. Table I lists the structural parameters of the prototype. Fig. 5 shows the current distribution results for each layer. It can be seen that the results calculated by the new Preisach type model are similar to those presented in [6]. The detail results will be shown in the full paper.

TABLE I
STRUCTURAL PARAMETERS OF SUPERCONDUCTING CABLE PROTOTYPE

No. of layer	r_i (mm)	θ_i (°)	Orientation
1	19.88	25	+1
2	20.60	10	+1
3	21.32	13	-1
4	22.07	36	-1

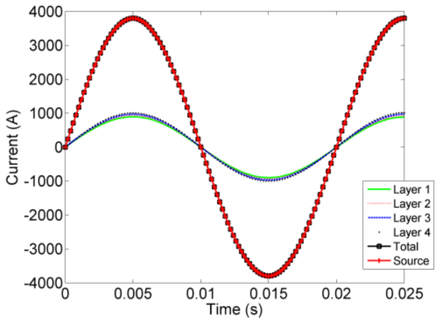


Fig. 5. Current distribution.

V.CONCLUSION

This paper presents a new Preisach type HTS hysteresis model which requires only the limiting hysteresis loop as the input data. The circuit model of HTS cable considering hysteresis is deduced. By comparison with the results in literatures, the correction of this model and circuit model is verified.

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